



Exercise 12F

1 Find the equation of the tangent to the curve:

a $y = x^2 - 7x + 10$ at the point $(2, 0)$

b $y = x + \frac{1}{x}$ at the point $(2, 2\frac{1}{2})$

c $y = 4\sqrt{x}$ at the point $(9, 12)$

d $y = \frac{2x-1}{x}$ at the point $(1, 1)$

e $y = 2x^3 + 6x + 10$ at the point $(-1, 2)$

f $y = x^2 - \frac{7}{x^2}$ at the point $(1, -6)$

2 Find the equation of the normal to the curve:

a $y = x^2 - 5x$ at the point $(6, 6)$

b $y = x^2 - \frac{8}{\sqrt{x}}$ at the point $(4, 12)$

(P) 3 Find the coordinates of the point where the tangent to the curve $y = x^2 + 1$ at the point $(2, 5)$ meets the normal to the same curve at the point $(1, 2)$.

(P) 4 Find the equations of the normals to the curve $y = x + x^3$ at the points $(0, 0)$ and $(1, 2)$, and find the coordinates of the point where these normals meet.

(P) 5 For $f(x) = 12 - 4x + 2x^2$, find the equations of the tangent and the normal at the point where $x = -1$ on the curve with equation $y = f(x)$.

(E/P) 6 The point P with x -coordinate $\frac{1}{2}$ lies on the curve with equation $y = 2x^2$.

The normal to the curve at P intersects the curve at points P and Q .

Find the coordinates of Q . (6 marks)

Problem-solving

Draw a sketch showing the curve, the point P and the normal. This will help you check that your answer makes sense.

Challenge

The line L is a tangent to the curve with equation $y = 4x^2 + 1$. L cuts the y -axis at $(0, -8)$ and has a positive gradient. Find the equation of L in the form $y = mx + c$.

Hint

Use the discriminant to find the value of m when the line just touches the curve. ← Section 2.5

Exam Question

12 A curve has equation $y = 6x\sqrt{x} + \frac{32}{x}$ for $x > 0$

12 (a) Find $\frac{dy}{dx}$

[4 marks]

12 (b) The point A lies on the curve and has x -coordinate 4

Find the coordinates of the point where the tangent to the curve at A crosses the x -axis.

[5 marks]

Extension Question

A curve has the equation $y = x^3 - px + q$.

The tangent to this curve at the point $(2, -8)$ is parallel to the x -axis.

a) Find the values of p and q .

b) Find also the coordinates of the other point where the tangent is parallel to the x -axis.